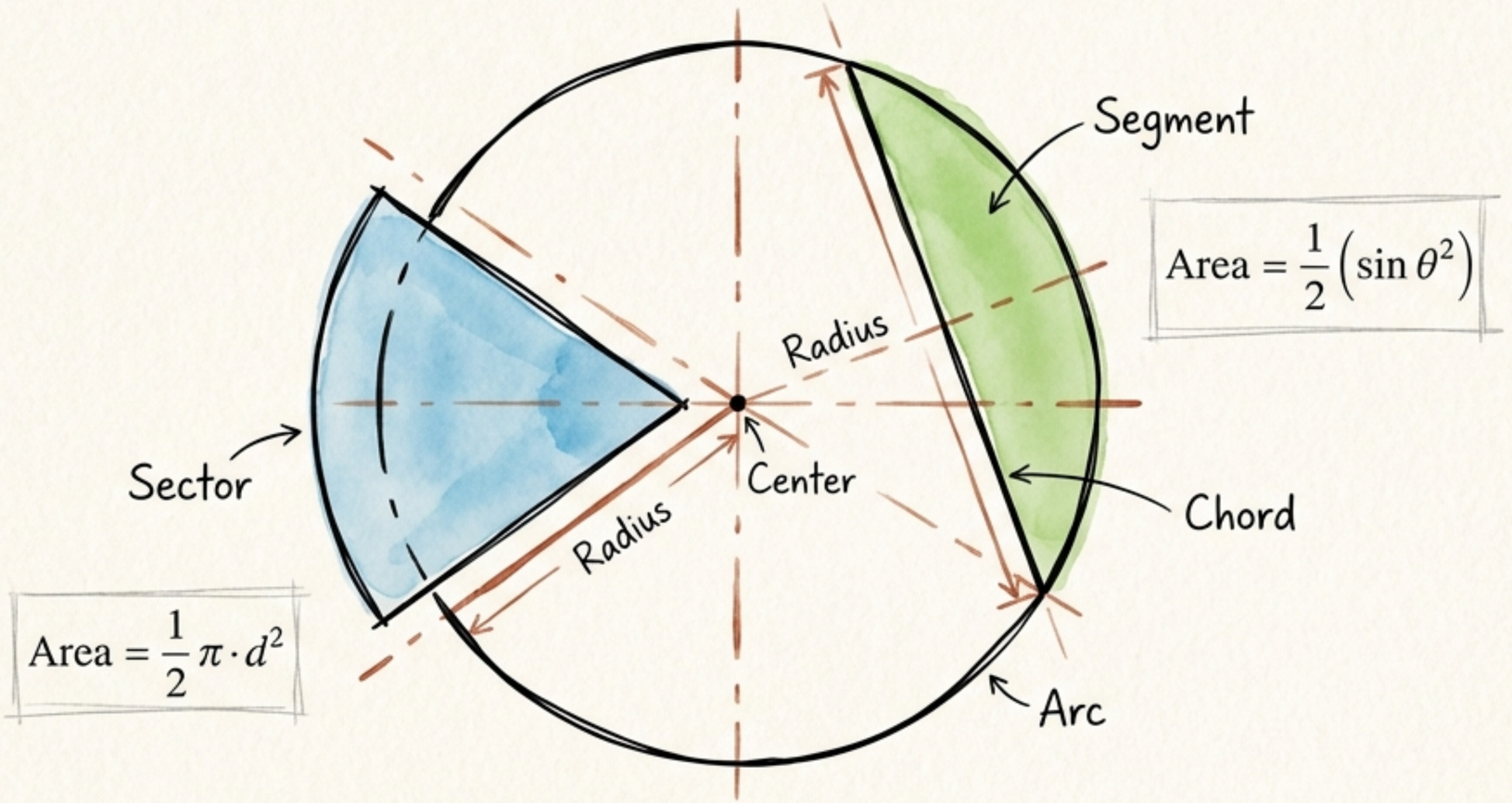
Areas Related to Circles

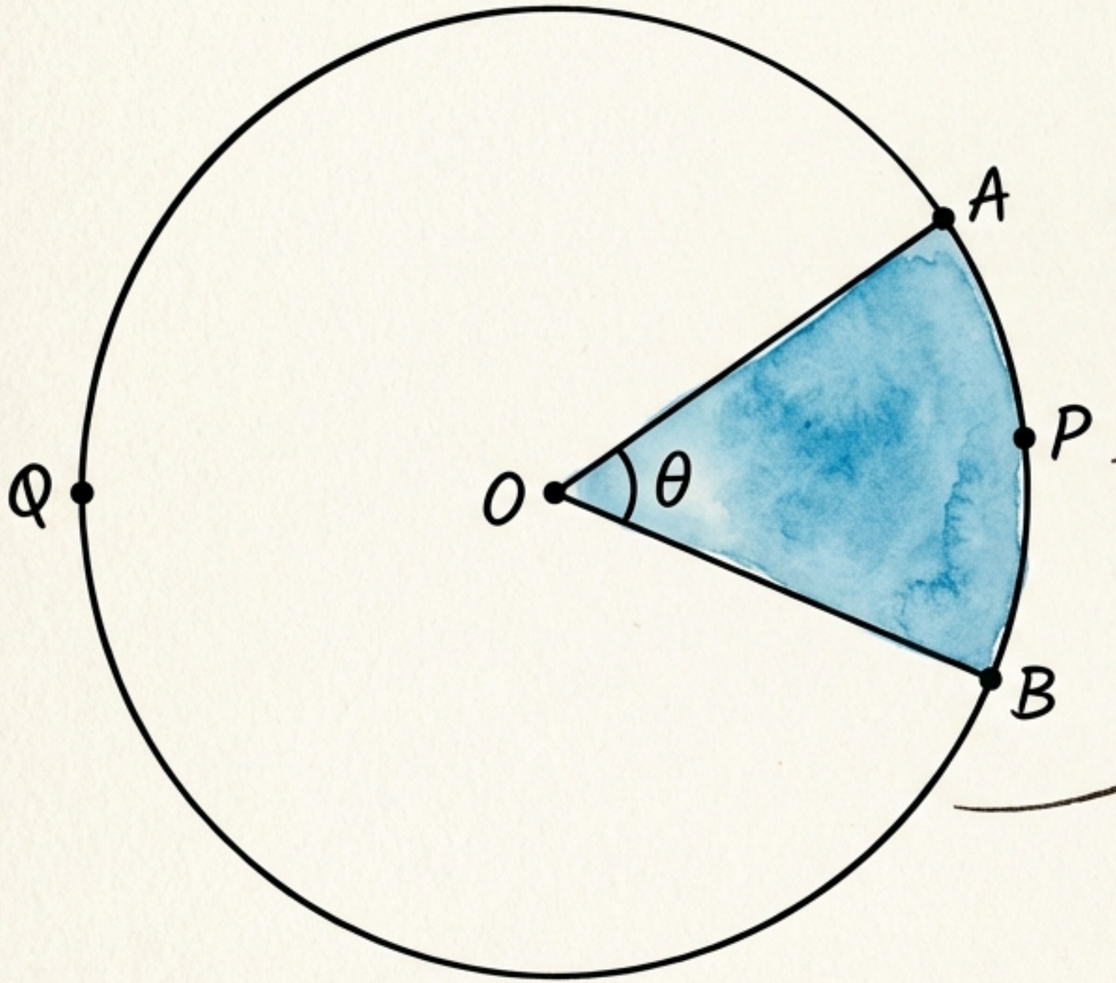


A Visual Guide to Sector and Segment (Chapter 11.1)

Based on NCERT Mathematics

NotebookLM

Concept: The Sector



The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle.

- *Minor Sector:* The shaded region *OAPB* (where angle < 180°).
- *Major Sector:* The unshaded region *OAQB* (angle is 360° - θ).

Note: When we write 'sector', we will mean the 'minor sector' unless stated otherwise.

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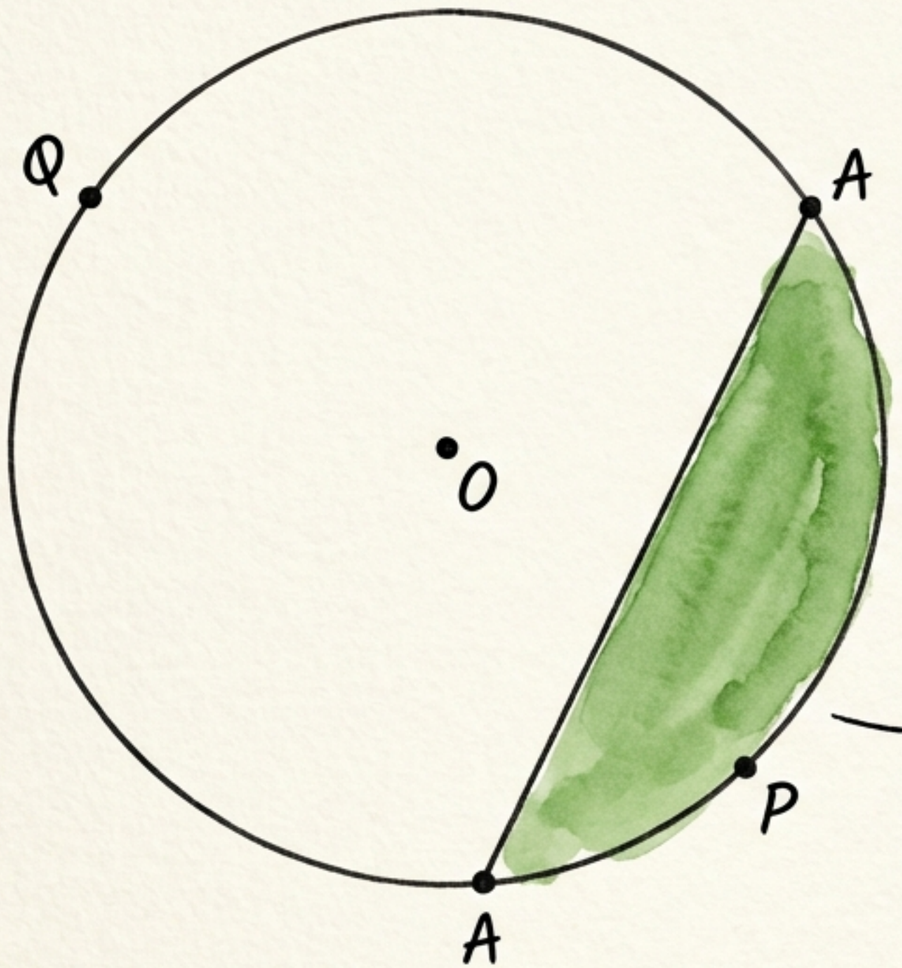
Concept: The Segment



The portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.

- **Minor Segment:** The shaded region *APB*.
- **Major Segment:** The unshaded region *AQB*.

> ***Note:*** *When we write 'segment', we will mean the 'minor segment' unless stated otherwise.*

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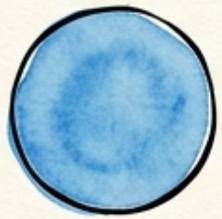
The Logic: Area of a Sector

Step 1

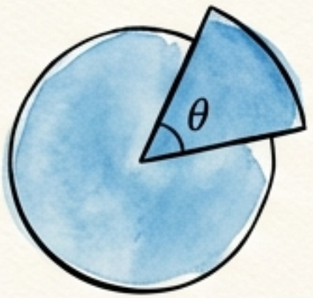


Angle $360° \rightarrow$ Area $= \pi r^2$

Step 2

Angle $1° \rightarrow$ Area $= \frac{\pi r^2}{360}$

Step 3



Angle $\theta \rightarrow$ Area $= \frac{\pi r^2}{360} \times \theta$

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

(where r is the radius and θ is the angle in degrees).

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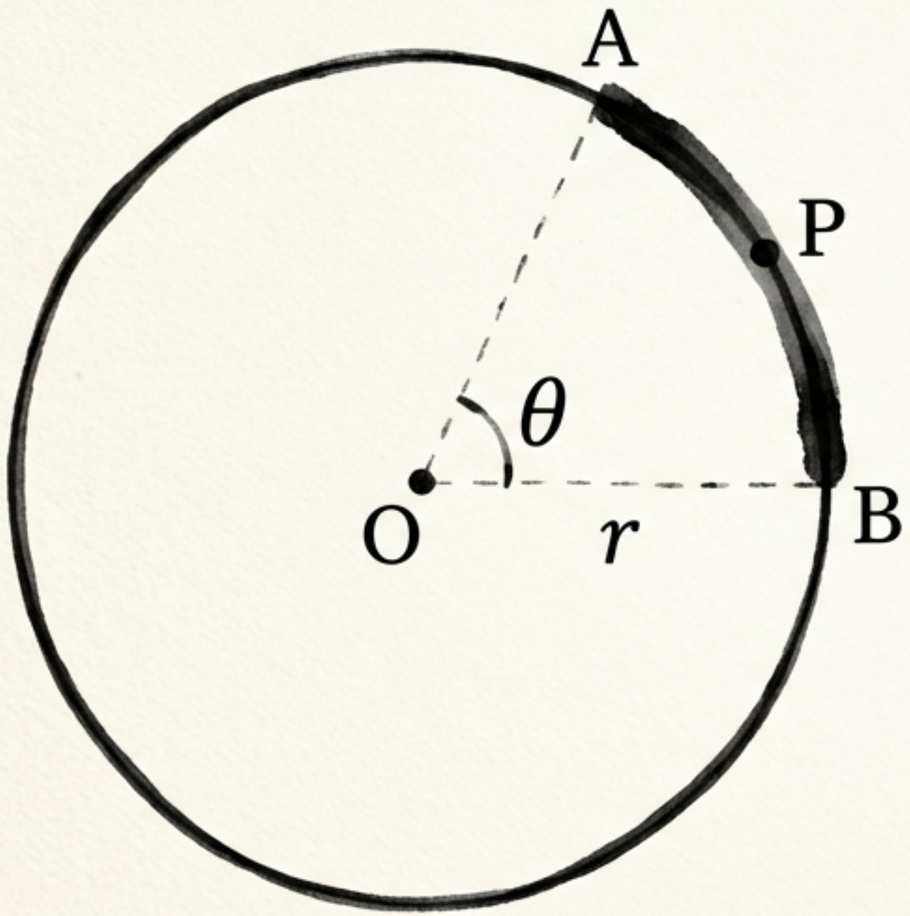
The Logic: Length of an Arc



Question: Can we find the length of the arc APB?

Apply the Unitary Method to the perimeter:

- Total length of circle (360°) = $2\pi r$
- Length for angle 1° = $\frac{2\pi r}{360}$
- Length for angle $\theta = \frac{\theta}{360} \times 2\pi r$

$$\text{Length of Arc} = \frac{\theta}{360} \times 2\pi r$$

(where r is the radius and θ is the angle in degrees).

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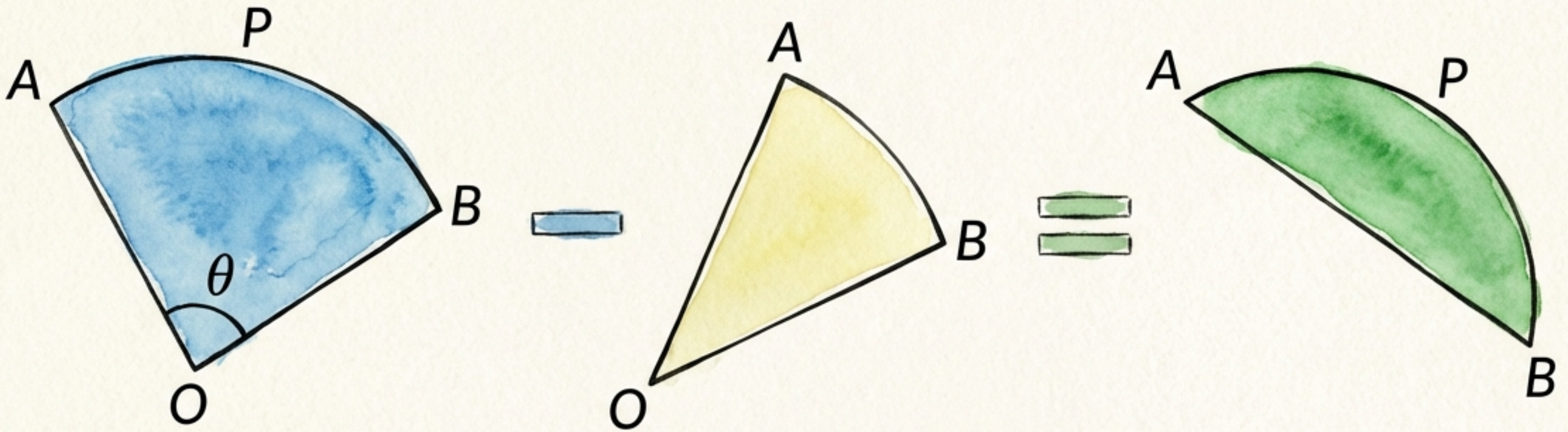
The Logic: Area of a Segment



Equation in Pictures

Area of Segment APB = Area of Sector OAPB − Area of Triangle OAB

Formula: $\left[\frac{\theta}{360} \times \pi r^2\right]$ − Area of ΔOAB

Area of Major Segment
= Area of Circle −
Area of Minor Segment

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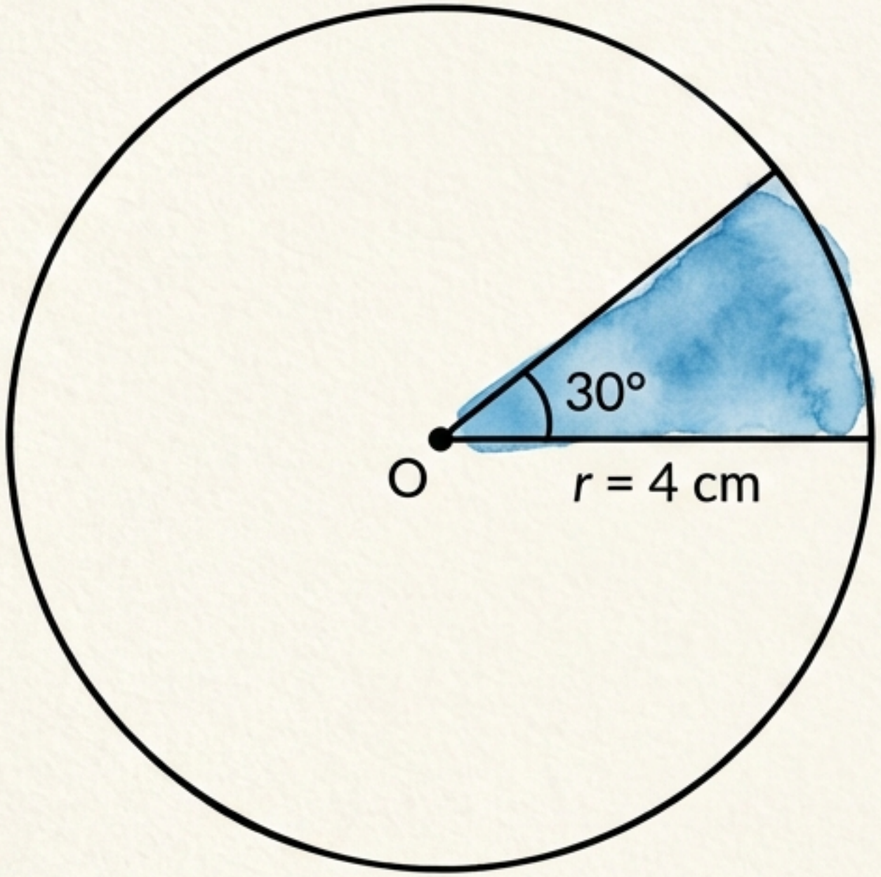
Worked Example 1: The Sector



Problem: Find the area of the sector with radius 4 cm and angle 30°. (Use $\pi = 3.14$)

Solution:

1. Minor Sector Area:

$$= \frac{30}{360} \times 3.14 \times 4 \times 4$$

$$= \frac{1}{12} \times 3.14 \times 16$$

$$\approx 4.19 \text{ cm}^2$$

2. Major Sector Area:

= Area of Circle – Area of Minor Sector

$$= (3.14 \times 16) - 4.19$$

$$\approx 46.1 \text{ cm}^2$$

(Alternative method: Area = $[(360\text{-}30)/360] \times \pi r^2$)

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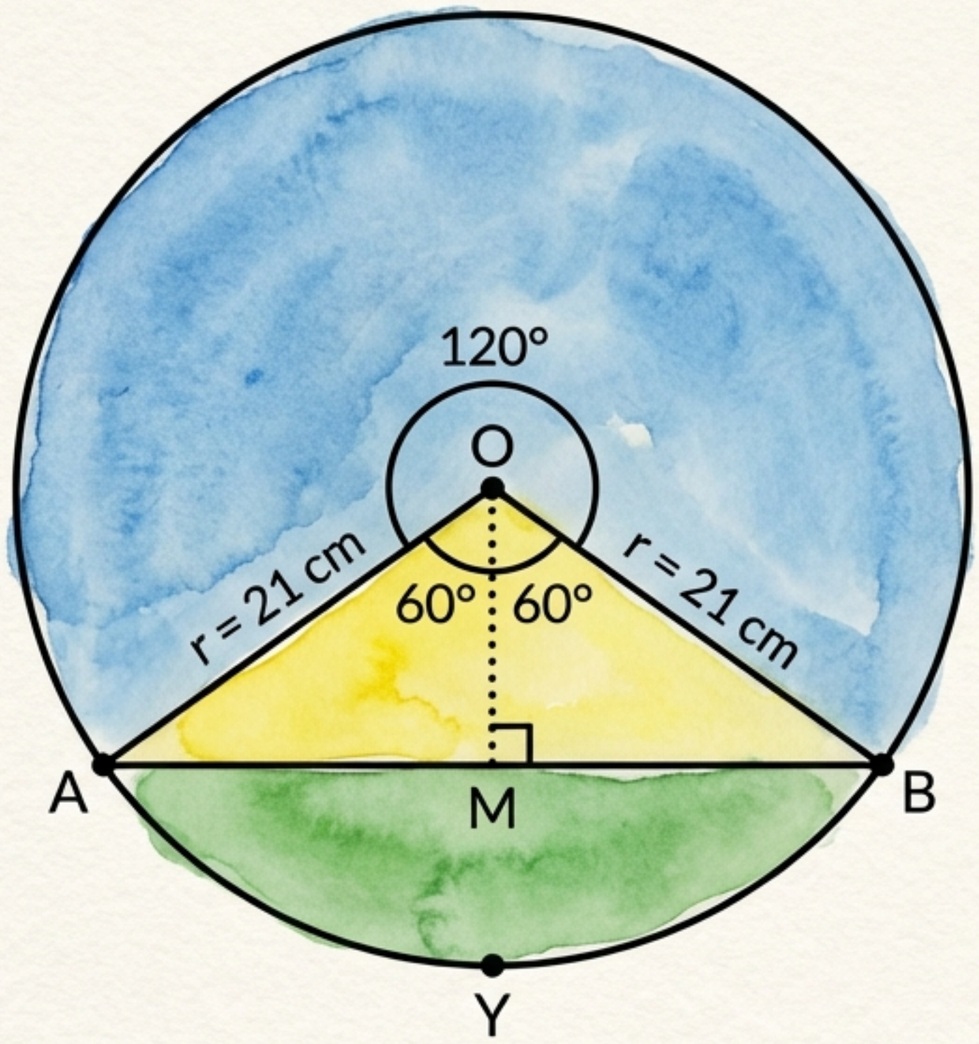
Worked Example 2: The Segment



Problem: Find area of segment AYB.
($r = 21$ cm, $\angle AOB = 120°, \pi = \frac{22}{7}$)

Step 1: Sector Area

$= \left(\frac{120}{360}\right) \times \frac{22}{7} \times 21 \times 21 = 462 \text{ cm}^2$ Patrick Hand

Step 2: Triangle Area (Using Trigonometry)

In ΔOMA (right-angled):

$OM = 21 \cos 60° = 21\left(\frac{1}{2}\right)$ cm Patrick Hand

$AM = 21 \sin 60° = 21\left(\frac{\sqrt{3}}{2}\right)$ cm

Base $AB = 2 \times AM = 21\sqrt{3}$ cm

Area $\Delta OAB = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} = \frac{441\sqrt{3}}{4} \text{ cm}^2$

Step 3: Final Subtraction

Segment Area $= 462 - \frac{441\sqrt{3}}{4} \approx \frac{21}{4}(88 - 21\sqrt{3}) \text{ cm}^2$

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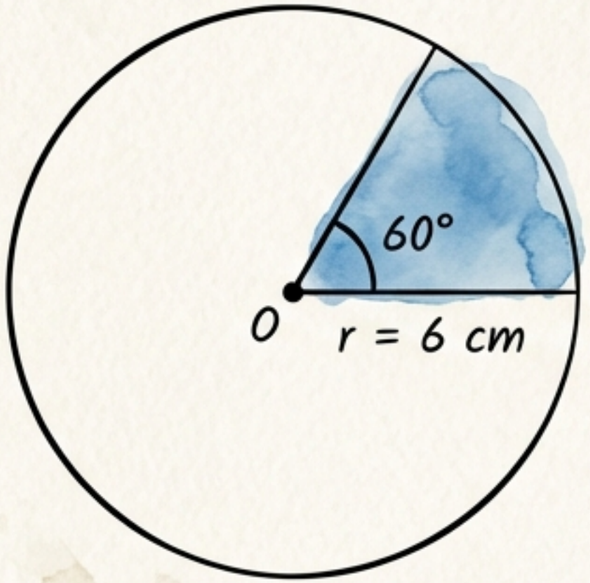
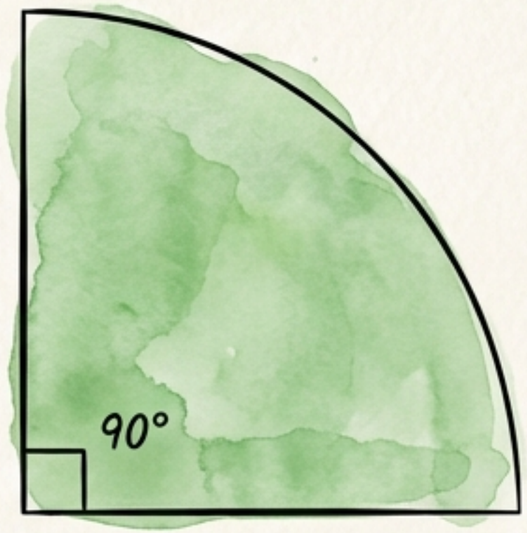
Exercise Gallery: The Basics



1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.



2. Find the area of a quadrant of a circle whose circumference is 22 cm.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

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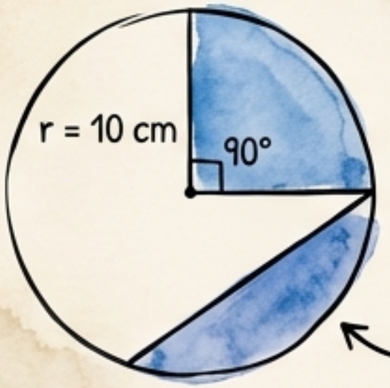
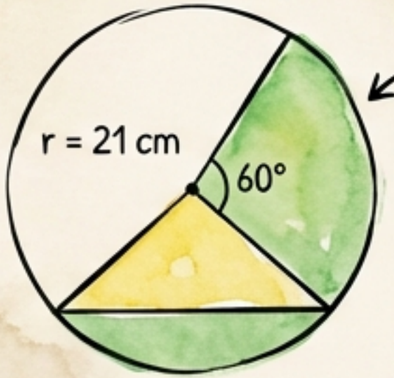
Exercise Gallery: Chords & Angles

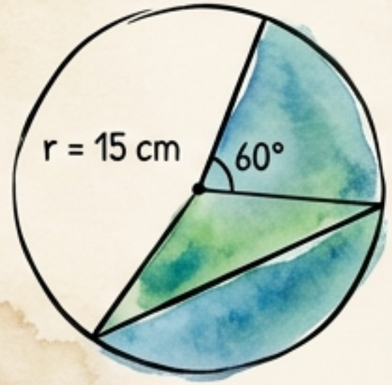


4. Radius 10 cm, Right Angle (90°) at center. Find area of:
(i) Minor Segment
(ii) Major Sector.

(Use π = 3.14)

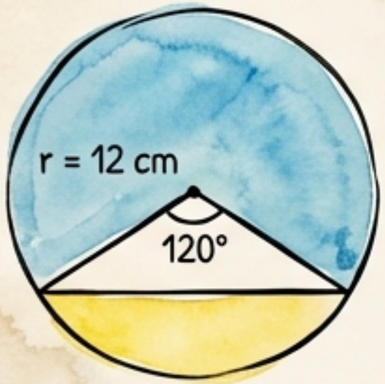


5. Radius 21 cm, Angle 60°. Find:
(i) Arc Length
(ii) Sector Area
(iii) Segment Area.



6. Radius 15 cm, Angle 60°. Find areas of corresponding Minor and Major Segments.

(Use π = 3.14, $\sqrt{3}$ = 1.73)



7. Radius 12 cm, Angle 120°. Find area of corresponding Segment.

(Use π = 3.14, $\sqrt{3}$ = 1.73)

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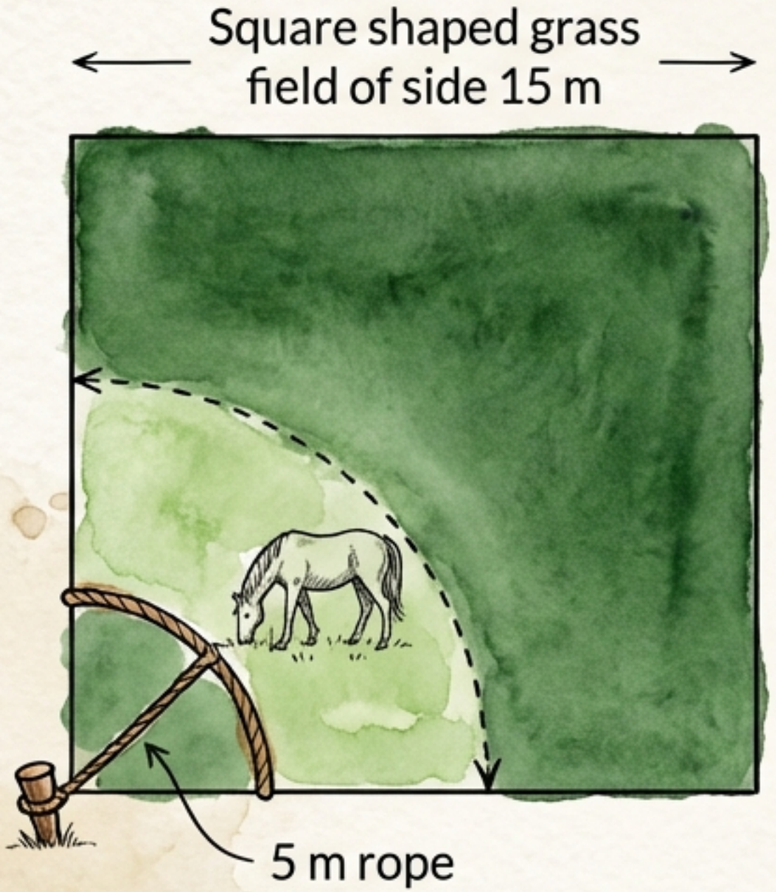
Feature Problem: The Horse



8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m.

Task (i): Find the area of that part of the field field in which the horse can graze (with a 5 m rope).

Task (ii): Find the increase in the grazing area if the rope were 10 m long instead of 5 m.

Note: Use π = 3.14

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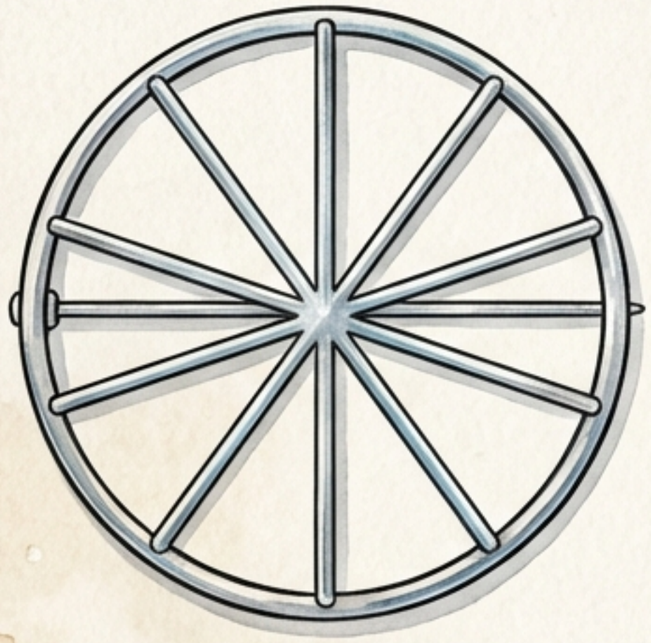
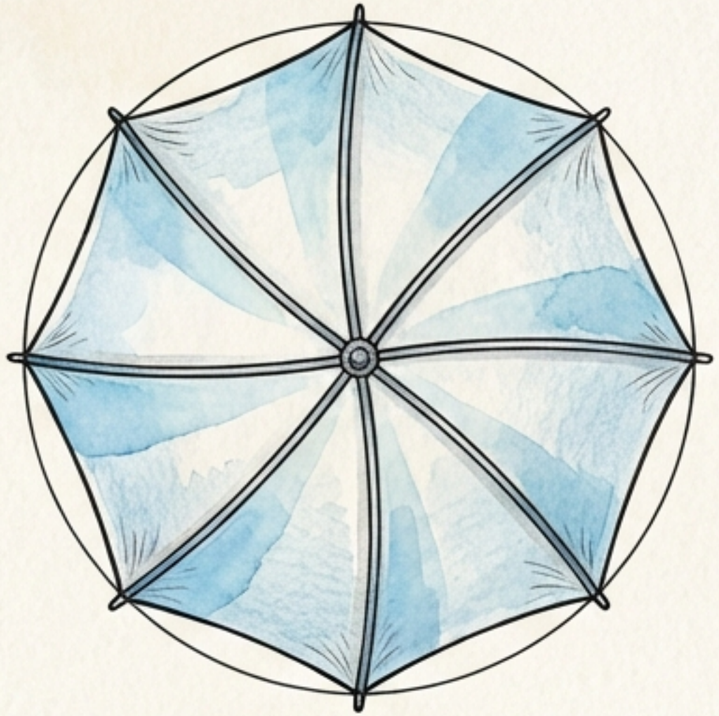
Applied Circles: Design & Mechanics



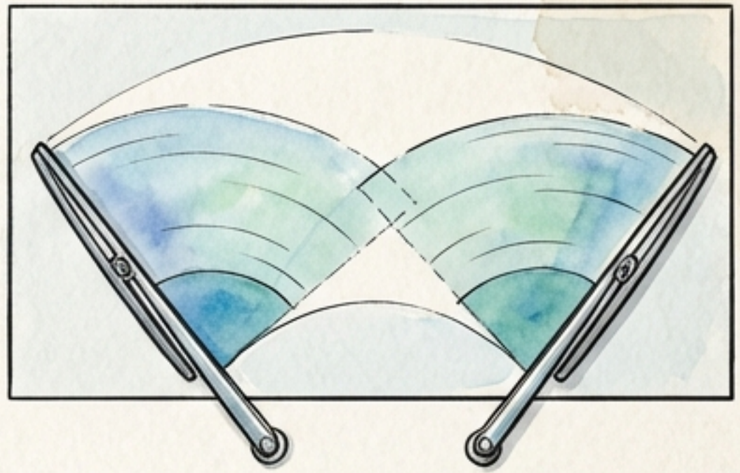
The Brooch

9. Silver Brooch: Diameter 35 mm. Wire forms circle + 5 diameters. Find (i) Total wire length (ii) Area of each sector.



The Umbrella

10. Umbrella: Radius 45 cm. 8 ribs. Find the area between two consecutive ribs.



The Wipers

11. Car Wipers: Two blades, length 25 cm, sweep angle 115°. Find total area cleaned at each sweep.

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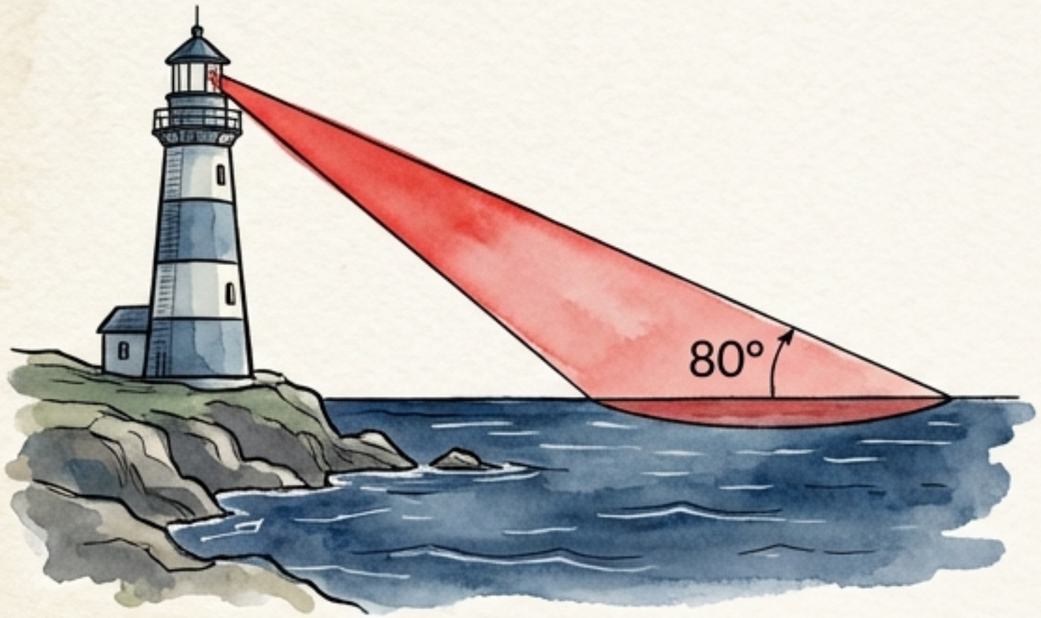
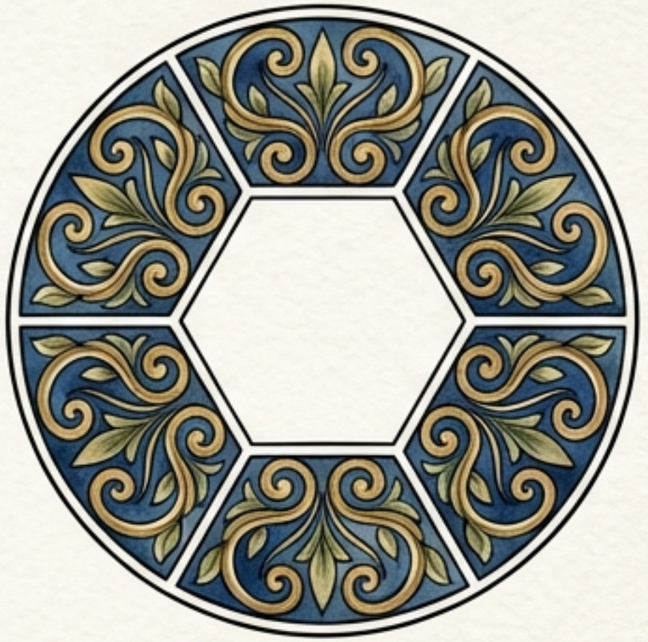
Applied Circles: Warnings & Decor

Warnings



12. Lighthouse: Spreads red light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea warning zone. (Use π = 3.14)

Decor



13. Round Table Cover: Radius 28 cm. Has 6 equal designs (leaving a central hexagon). Find the cost of making the designs at rate ₹0.35 per cm^2. (Use $\sqrt{3} = 1.7$)

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Summary & Theory

Key Formulas Recap:

- Arc Length = $(\theta / 360) \times 2\pi r$
- Sector Area = $(\theta / 360) \times \pi r^2$
- Segment Area = Sector Area – Triangle Area

14. Tick the correct answer: Area of a sector of angle p (in degrees) of a circle with radius R is:

(D) $\frac{p}{720} \times 2\pi R^2$ ← *Simplifies to* $(p/360) \times \pi R^2$

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End of Chapter 11.1

Playfair Display

Areas Related to Circles

Content based on NCERT Textbooks.

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